Windmill Primary Federation

Progression in Written Calculation Methods

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.

Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children need to be equipped with mental, written and calculator methods that they understand and can use correctly.

When faced with a calculation, children should be able to decide which method is most appropriate and will have strategies that can be used to check its accuracy. They will do this by asking themselves:

- Can I do this in my head?
- Can I do this in my head using a drawing or jottings?
- Do I need to use a pencil and paper procedure?
- Do I need a calculator?

At every stage of the progression outlined within this guide, successful learning will need to be underpinned by a secure and appropriate knowledge of number facts, along with the mental skills that are needed to perform each process and judge if it was successful.

The overall aim is that when children leave The Windmill Primary Federation they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use there knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient and reliable written method of calculation for each operation that can be applied with confidence when undertaking calculations that cannot be carried out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Written methods for addition

To add successfully, children need to be able to:

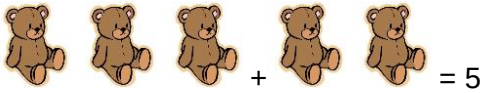
- recall all addition pairs to 9 + 9 and complements in 10 e.g. 6 + 4 = 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Stage 1

At first children will relate addition to the combining of 2 groups:

For example: 3 + 2 = 5

Count out 3, count out 2. Put together and count out 5



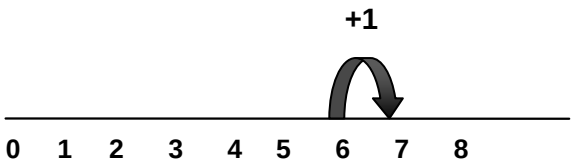
Children are encouraged to make use of fingers as these are a constantly available resource for calculations at this level.

Alternatively, count out 3 and then count on 2 more to make 5

Stage 2

The next step is to be able to count one more, and then *several more*, on a number line:

For example: 6 + 1 = 7



Children need to understand the concept of the '=' sign, so that it is not just interpreted as the answer. For this reason, calculations should be written on either side of the '='.

For example: 7 = 6 + 1 and 6 + 1 = 7

Calculations should be presented where missing numbers are in a variety of places and should include the addition of numbers up to 20.

For example: ☐ + 1 = 7 and 6 + ☐ = 7

(≈ Year 1)

Stage 3

Steps in addition can be recorded on an empty number line. The steps often bridge through a multiple of 10.

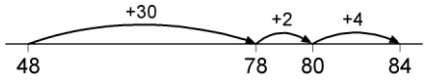
Mental methods involve partitioning, e.g. adding the tens and units separately, often starting with the tens.

The empty number line helps to record the steps on the way to calculating the total.

(≈ Year 2)

8 + 7 = 15

48 + 36 = 84



or:

This method can be used for the addition of larger numbers and decimals further up the school.

35.8 + 7.3 = 43.1

(≈ Year 5)

Stage 4

The next stage is to record steps in addition using partitioning (Horizontal Expansion)

Add the tens and then the units to form partial sums and then add these partial sums.

Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sums either the tens or the units can be added first, and the total of the partial sums can be found by adding them in any order.

The addition of the tens in the calculation 83 + 42 is described in the words 'eighty plus forty equals one hundred and twenty', stressing the link to the related fact 'eight plus four equals twelve'

(≈ Year 4)

83 + 42

```
  80 + 3
  40 + 2
------------
120 + 5   = 125
```

Where used a method to support the calculation of HTU + HTU, the written method should be presented as follows.

367 + 185

```
  300 +  60 +  7
  100 +  80 +  5
--------------------
400 + 140 + 12   = 552
```

Stage 5

Vertical expansion

367 + 185

```
   367
   185
--------
Units      12   (7+5)
Tens      140   (60 + 80)
Hundreds  400   (300+100)
---------
   552
```

(≈ Year 5)

Stage 6

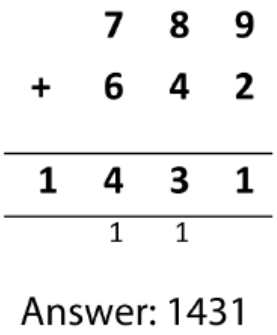
In this method, recording is reduced further. Carried digits are recorded below the line, when teaching, pupils should be encouraged to use the appropriate mathematical language: 'carry ten' or 'carry one hundred', not 'carry one'.

Later, this method can be extended to include the addition of three two digit numbers, two three digit numbers and numbers with different numbers of digits.

8 tenths add 6 tenths makes 14 tenths, or 1 whole and 4 tenths. The 1 whole is 'carried' into the units column and the 4 tenths is written in the tenths column.

Extend to numbers with any number of digits and decimals with 2 and 3 decimal places.

(≈ Year 5/6)

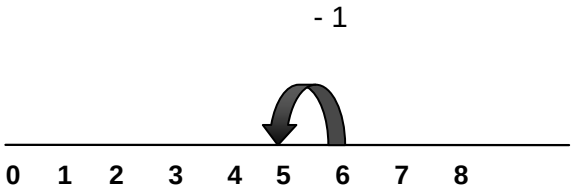
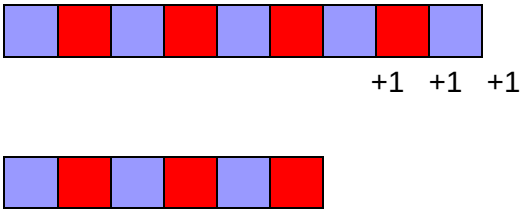


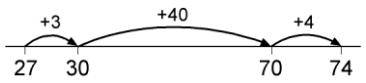
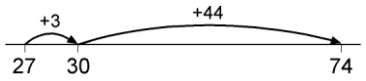
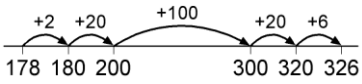
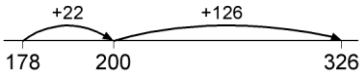
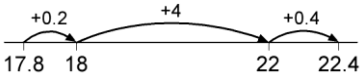
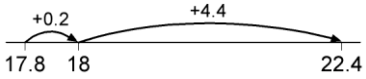
Answer: 1431

Written methods for subtraction

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into hundreds, tens and units in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

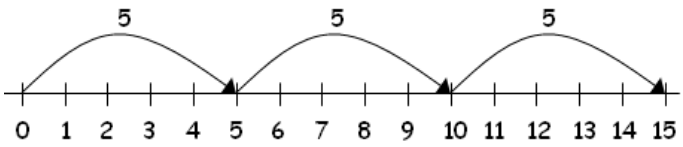
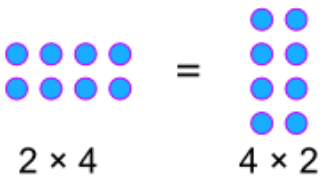
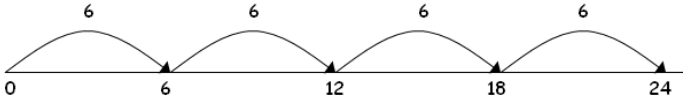
Stage 1	
In the early stages, children will be taught to 'take away' one or two objects and find the new total.	For example: 5-3=2  5 take 2 away is 3 
Stage 2	
The next stage is for children to be able to work out one less or several less on a number line	For example: 6 – 1 = 5 
Stage 3	
At an early stage children are introduced to the concept of difference. Children should be taught that subtraction can be worked out by counting on to find the difference between two numbers. Calculations should include the subtraction of numbers up to 20. (≈ Year 1)	For example: How much longer is this row of cubes than this one? 9 – 3 = 6 

Stage 4	
The counting up method The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally. (≈ Year 2/3)	 74 − 27 3 → 30 40 → 70 4 → 74 47 or:  74 − 27 3 → 30 44 → 74 47
Stage 5	
With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally. The most compact form of recording remains reasonably efficient. (≈ Year 4)	 326 −178 2 → 180 20 → 200 100 → 300 26 → 326 148 or:  326 −178 22 → 200 126 → 326 148
Stage 6	
The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. This counting-up method is a useful alternative to decomposition for children whose struggle with decomposition. (≈ Year 5)	 22.4 −17.8 0.2 → 18 4.0 → 22 0.4 → 22.4 4.6 or:  22.4 −17.8 0.2 → 18 4.4 → 22.4 4.6
Stage 7	
The decomposition method of subtraction. (≈ Year 5/6)	8 12 1 9 3 2 − 4 5 7 4 7 5 Answer: 475

Written methods for multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to 12 × 12 by the end of Year Four;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers

Stage 1	
First children are taught to count in 2s, 10s and 5s using practical objects.	For example: 3 x 2 = 6  3 x 5 = 15  = 15
Stage 2	
Repeated addition on number lines 3 times 5 is 5 + 5 + 5 = 15 Repeated addition can be shown easily on a number line. Repeated addition using arrays, this knowledge will help to support the development of the grid method and will help children understand that calculations can be done either way, i.e. 2 x 4 or 4 x 2, according to the commutative law. Repeated addition on a blank number line. 4 times 6 is the same as 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4	 a × b = b × a Example:  2 × 4 4 × 2 

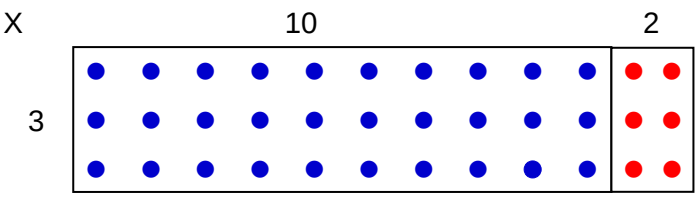
Stage 3

Children will be taught the progression from the use of arrays to the more formal written method of multiplication, using a grid.

Children need a secure knowledge of the effect of multiplying a single digit by ten.

(≈ Year 2)

12 x 3



X	10	2
3	30	6

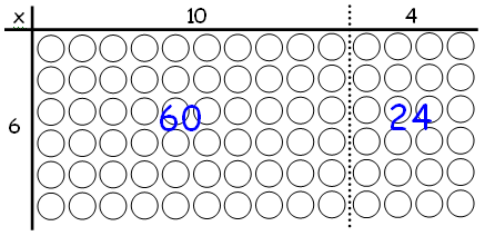
= 30 + 6 = 36

Stage 4

Children will continue to use arrays…

…where appropriate leading onto more complex examples of the grid method of multiplication that were introduced in stage 3.

(≈ Year 3)



(6 x 10) + (6 x 4)

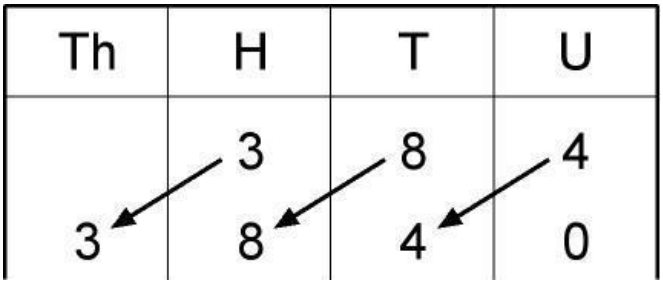
60 + 24

84

Stage 5

Children are taught that, when multiplying by multiples of 10, they are not simply 'adding zeros'. It is essential that they understand that the '0' is called a 'place holder' and is added after the digits have been moved to the left (one place to the left when multiplying by 10, two places to the left when multiplying by 100 and so on).

384 x 10



Stage 6

Grid method for TU x U (children should try to approximate the answer first.)

(≈ Year 3/4)

23 X 8

X	20	3
7	140	21

140 + 21 = 161

Grid method for HTU x U (children should try to approximate the answer first.)

(≈ Year 4)

346 X 9

X	300	40	6
9	2700	360	54

(By the end of Year 4, children will be able to use an efficient method for three digit by one digit multiplication)

2700 + 360 + 54

2700 + 360 = 3060

3060 + 54 = 3114

Stage 7

Extend to TU × TU (children should try to approximate the answer first.)

Start with the grid method. The partial products in each row are added (1000 + 120 = 1120 and 350 + 42 = 392), and then the two sums at the end of each row are added to find the total product (1120 + 392 = 1512).

(This method should be extended to incorporate simple decimals as appropriate)

(≈ Year 4/5)

56 × 27 is approximately 60 × 30 = 1800.

X	**50**	**6**
20	1000	120
7	350	42

Please refer to the addition section of this handbook for more detail on how to find the total (1512)

Stage 8

When they are ready and not before, children should be taught vertical multiplication.

Expanded Vertical Method

38 x 7

```
   38
 X  7
------
   56    (8 x 7)
+ 210    (30 x 7)
--------
  266
```

56 x 27

```
   56
 X 27
--------
   42    (6  X  7)
  350    (50  X  7)
  120    (6  X 20)
+ 1000   (50 X 20)
--------
 1512
```

(≈ Year 5/6)

Stage 9

Compact Vertical Method

```
  3 4 2
×     7
-------
2 3 9 4
  2 1
```

Leading to…

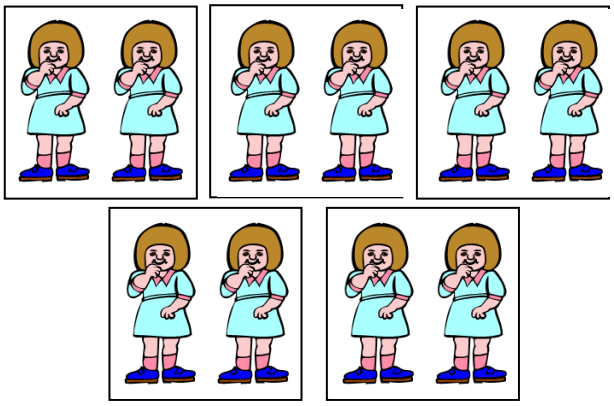
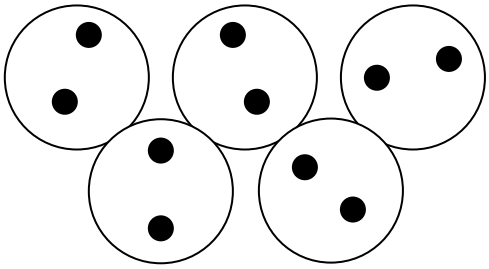
```
  1 2 4
×   2 6
-------
  7 4 4
  1 2
2 4 8 0
-------
3 2 2 4
1 1
```

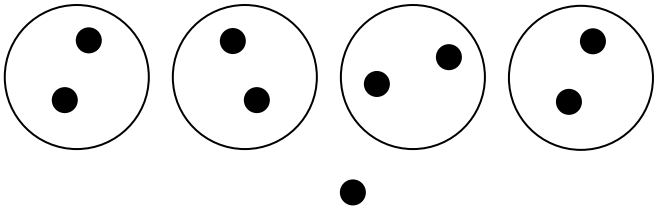
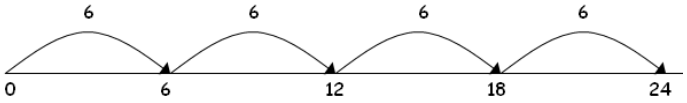
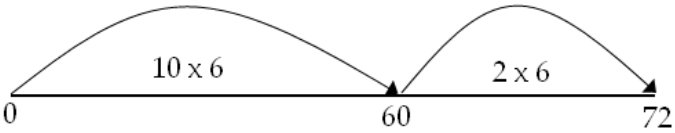
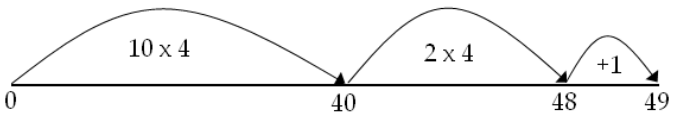
(≈ Year 6)

Written methods for division

To divide successfully, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into hundreds, tens and units in different ways;
- recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.
- understand division as repeated addition;
- multiply a two-digit number by a single-digit number mentally;
- add numbers using an appropriate method.

Stage 1	
Children will be given the opportunity to share objects into equal groups, counting how many objects there are in each group. Understanding division as sharing.	6 cakes are shared between 2 people. How many cakes does each receive? 
Stage 2	
Understanding division as grouping. Children will be given the opportunity to sort objects into 2s, 3s, 4s etc. (≈ Year 1)	10 children are grouped into teams of 2. How many teams will there be?  10 ÷ 2 = 5 As a jotting this could look like… 

Stage 3	
Children will become more familiar with the use of practical and informal written methods and the related vocabulary to support division, including calculations with remainders. (≈ Year 2)	9 ÷ 2 = 4 r 1 As a jotting this could look like… 
Stage 4	
Counting on a number line An empty number line can be used to record grouping or sharing. (≈ Year 2)	For example: 24 ÷ 6 = 4  This could be 24 chocolate eggs that are to be shared between 6 children OR 24 chocolate eggs that are to be packed (grouped) in boxes of 6
Stage 5	
Children will use their knowledge of multiplication facts to divide on an empty number line. (≈ Year 3) *NB This method should only be used once the children are able to confidently use multiplication facts.*	For example: 72 ÷ 6 = 12  10 groups of 6 + 2 groups of 6 = 12 groups of 6
Stage 6	
Remainders 49 ÷ 4 = 12 r1 Grouping - How many 4's make 49, how many units are left over? Sharing - 49 shared between 4, how many units are left over? (≈ Year 3)	For example: 49 ÷ 4 = 12 r 1  10 groups of 4 + 2 groups of 4 with one unit left over

Stage 7	
Grouping Method (≈ Year 4/5)	How many packs of 3 can we make from 72 biscuits? 3) 72 - 30 — 10 groups of 3 42 - 30 — 10 groups of 3 12 - 12 — 4 groups of 3 0 Answer 24 Short division can also be used to complete calculations where there is a remainder. 32 r 4 6) 196 - 180 — 30 groups of 6 16 - 12 — 2 groups of 6 4 Answer : 32 remainder 4 *or* 32 r 4
Stage 8	
Compact 'Bus Stop' Method (≈ Year 5/6)	98 ÷ 7 becomes 1 4 7 | 9 (2) 8 Answer: 14 432 ÷ 5 becomes 8 6 r 2 5 | 4 3 (3) 2 Answer: 86 remainder 2 Within the above example, the children should be taught to express the quotient as a fraction (86 $^{2}/_{5}$) and a decimal (86.4).

Stage 9	
Long division will only be taught to those children who are confident in the above methods. (≈ Year 5/6)	27 36) 972 - 720 20 groups of 36 252 - 252 7 groups of 36 0 Answer 27
Stage 10	
(≈ Year 6)	432 ÷ 15 becomes **2 8** **1 5** | **4 3 2** **3 0 0** 15×20 **1 3 2** **1 2 0** 15×8 **1 2** $\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ or 28.8